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MTH 530, Abstract Algebra I (graduate) Fall 2012 ,HW number THREE (Due: Sat. at 1pm October 20)

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QUESTION 1. (i) Let G be an Abelian group with an odd number of elements. Prove that the product of all elements of G is the identity.

(ii) Let N be a normal subgroup of a group $(G, *)$. If H is a subgroup of G, then prove that $N * H = \{n * h \mid n \in N$ and $h \in H\}$ is a subgroup of G.

(iii) Give me an example of a group $(G, *)$ such that G has two subgroups H and N such that $|N| = |H| = 2$ but $H * N$ is not a subgroup of G.

(iv) Let N, H be normal subgroups of a group $(G, *)$. Prove that $N * H = \{n * h \mid n \in N$ and $h \in H\}$ is a normal subgroup of G.

(v) Given $H = A_3 \oplus \{0, 3\}$ is a subgroup of the non-abelian group $G = S_3 \oplus Z_6$. Find all distinct left and right cosets of H inside G. Can we conclude that H is a normal subgroup of G? (the answer should be yes). Hence G/H is a group. Prove that G/H is a cyclic group and hence abelian.

(vi) I told you that $|A_n| = n!/2$ (n > 1). Now let us prove it. You know that A_n is a subgroup of S_n, so let O_n be the set of all odd permutation of S_n. Show that $O_n = (1 \quad 2)oA_n$.

(vii) Let $a \in S_4$. Find all possibilities for $|a|$. Note that $6, 8, 12$ are factors of 24. So from your answer, is the following statement right? If G is a finite group of order n and $m \in Z^+$ such that $m|n$, then G has an element of order m.

(viii) Let $f = (2\ 4\ 5\ 1)o(4\ 3\ 5\ 1\ 2)$ find $|f|$.

Faculty information

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